

## C2 Exercise 4A (mid-point of segment)

Note Title

06/12/2011

1 Find the midpoint of the line joining...

a  $(4, 2)$  and  $(6, 8) = \left(\frac{4+6}{2}, \frac{2+8}{2}\right) = (5, 5)$  ✓

b  $(0, 6)$  and  $(12, 2) = (6, 4)$  ✓

c  $(2, 2)$  and  $(-4, 6) = (-1, 4)$  ✓

d  $(-6, 4)$  and  $(6, -4) = (0, 0)$  ✓

e  $(-5, 3)$  and  $(7, 5) = (1, 4)$  ✓

f  $(7, -4)$  and  $(-3, 6) = (2, 1)$  ✓

g  $(-5, -5)$  and  $(-11, 8) = (-8, 1\frac{1}{2})$  or  $(-8, \frac{3}{2})$  ✓

h  $(6a, 4b)$  and  $(2a, -4b) = \left(\frac{8a}{2}, 0\right) = (4a, 0)$  ✓

i  $(2p, -q)$  and  $(4p, 5q) = (3p, 2q)$  ✓

j  $(-2s, -7t)$  and  $(5s, t) = \left(\frac{3}{2}s, -3t\right)$  ✓

k  $(-4u, 0)$  and  $(3u, -2v) = \left(-\frac{1}{2}u, -v\right)$  ✓

l  $(a+b, 2a-b)$  and  $(3a-b, -b)$

$= \left(\frac{4a}{2}, \frac{2a-2b}{2}\right) = (2a, a-b)$  ✓

$$m \quad (4\sqrt{2}, 1), (2\sqrt{2}, 7)$$

$$= \left( \frac{6\sqrt{2}}{2}, \frac{8}{2} \right) = (3\sqrt{2}, 4) \checkmark$$

$$n \quad (-\sqrt{3}, 3\sqrt{5}) \text{ and } (5\sqrt{3}, 2\sqrt{5})$$

$$= \left( \frac{4\sqrt{3}}{2}, \frac{5\sqrt{5}}{2} \right) = (2\sqrt{3}, \frac{5\sqrt{5}}{2}) \checkmark$$

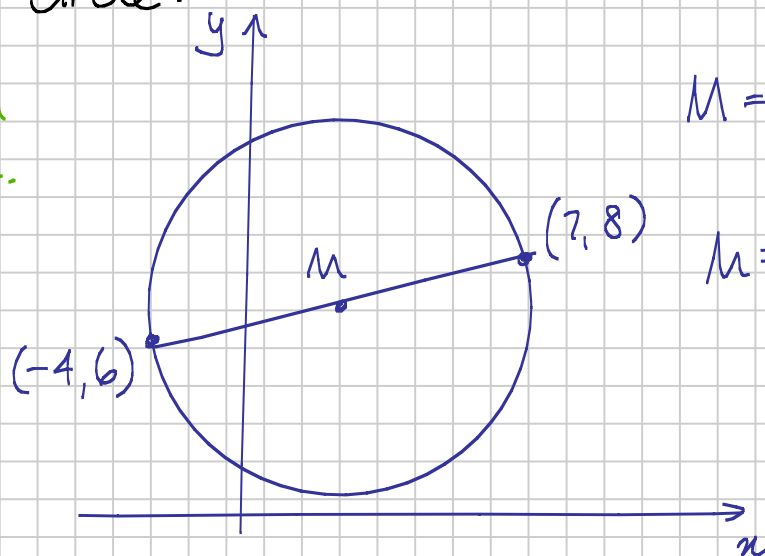
$$o \quad (\sqrt{2}-\sqrt{3}, 3\sqrt{2}+4\sqrt{3}) \text{ and } (3\sqrt{2}+\sqrt{3}, -\sqrt{2}+2\sqrt{3})$$

$$= \left( \frac{4\sqrt{2}}{2}, \frac{2\sqrt{2}+6\sqrt{3}}{2} \right)$$

$$= (2\sqrt{2}, \sqrt{2}+3\sqrt{3}) \checkmark$$

2 The line PQ is the diameter of a circle where P and Q are  $(-4, 6)$  and  $(7, 8)$  respectively. Find the coordinates of the centre of the circle.

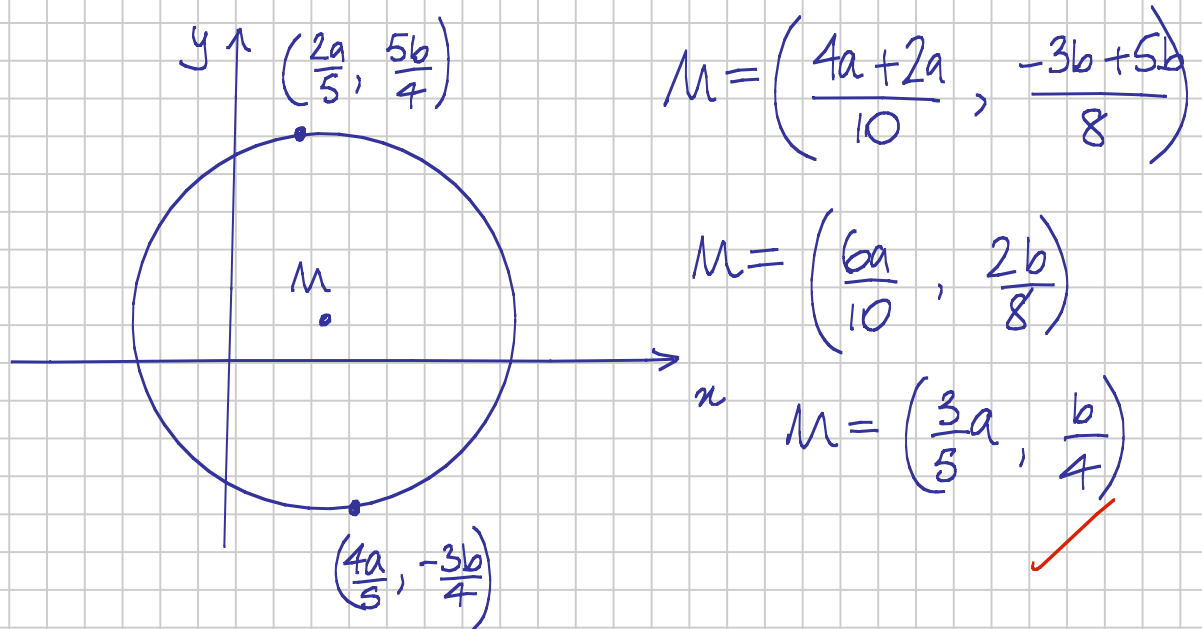
⚠ draw a diagram.



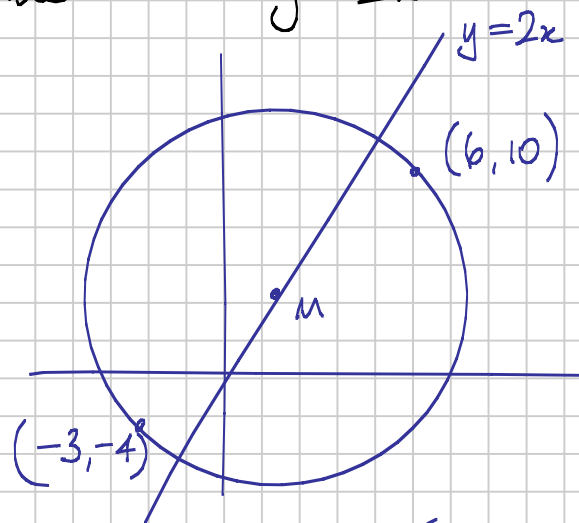
$$M = \left( \frac{-4+7}{2}, \frac{6+8}{2} \right)$$

$$M = \left( \frac{3}{2}, 7 \right) \checkmark$$

- 3 The line RS is a diameter of a circle, where R and S are  $\left(\frac{4a}{5}, -\frac{3b}{4}\right)$  and  $\left(\frac{2a}{5}, \frac{5b}{4}\right)$  respectively. Find the coordinates of the centre of the circle.



- 4 The line AB is a diameter of a circle where A and B are  $(-3, -4)$  and  $(6, 10)$  respectively. Show that the centre of the circle lies on the line  $y = 2x$ .

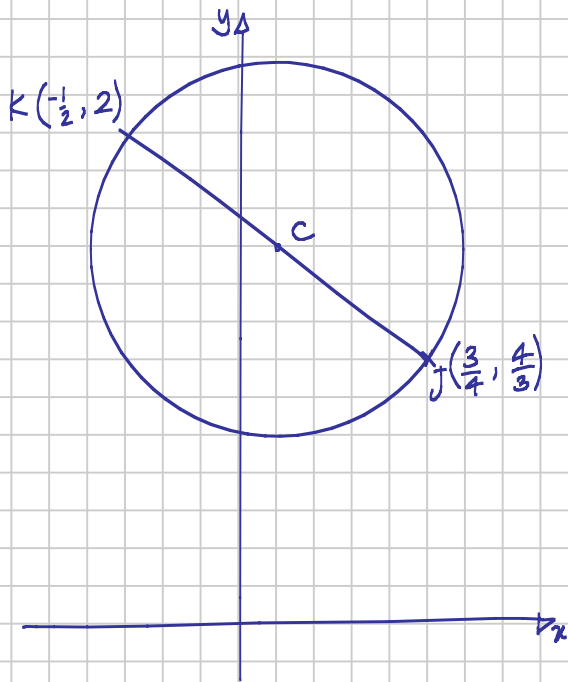


① find  $M$ .

$$M = \left(\frac{6+(-3)}{2}, \frac{10+(-4)}{2}\right)$$

$$M = \left(\frac{3}{2}, \frac{6}{2}\right)$$

5 The line JK is a diameter of a circle, where J and K are  $(\frac{3}{4}, \frac{4}{3})$  and  $(-\frac{1}{2}, 2)$  respectively. Show that the centre of the circle lies on the line  $y = 8x + \frac{2}{3}$ .



$$C = \left( \frac{\frac{3}{4} + (-\frac{1}{2})}{2}, \frac{\frac{4}{3} + 2}{2} \right)$$

$$C = \left( \frac{1}{8}, \frac{5}{3} \right)$$

$$\text{Put } x = \frac{1}{8} \text{ into } y = 8x + \frac{2}{3}$$

$$y = 8 \times \frac{1}{8} + \frac{2}{3}$$

$$y = 1\frac{2}{3} = \frac{5}{3}$$

hence C is on the line.

6 The line AB is a diameter of a circle, where A and B are  $(0, -2)$  and  $(6, -5)$  respectively. Show that the centre of the circle lies on the line  $x - 2y - 10 = 0$

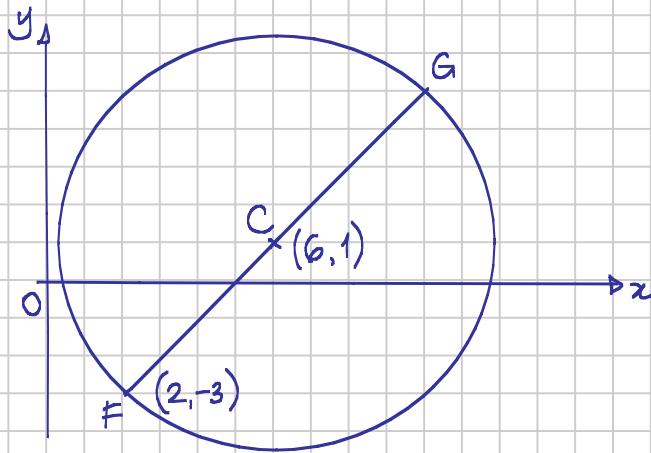
Centre C has coordinates  $\left(\frac{0+6}{2}, \frac{-2+(-5)}{2}\right) = (3, -3\frac{1}{2})$

Now put  $x = 3$ ,  $y = -3\frac{1}{2}$  in  $x - 2y - 10$

to get  $3 - 2(-3\frac{1}{2}) - 10 = 3 + 7 - 10 = 0$

So it satisfies the equation of the line.

- 7 The line  $FG$  is a diameter of the circle centre  $(6,1)$ . Given  $F$  is  $(2,-3)$ , find the coordinates of  $G$ .



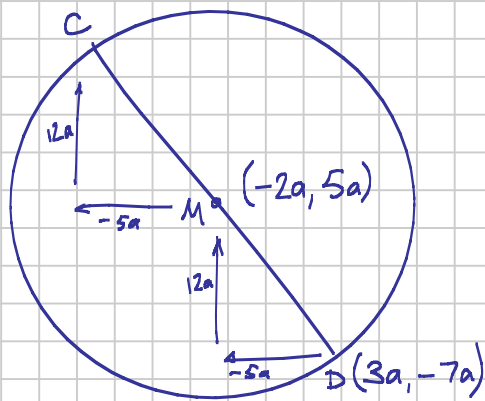
$$\overrightarrow{FC} = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$$

$$\text{so } \overrightarrow{CG} = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$$

Hence  $G$  is  $(10,5)$

The line segment (vector)  $FG$  is equal to the line segment  $CG$ .

- 8 The line  $CD$  is a diameter of the circle centre  $(-2a, 5a)$ . Given that  $D$  has coordinates  $(3a, -7a)$ , find the coordinates of  $C$ .

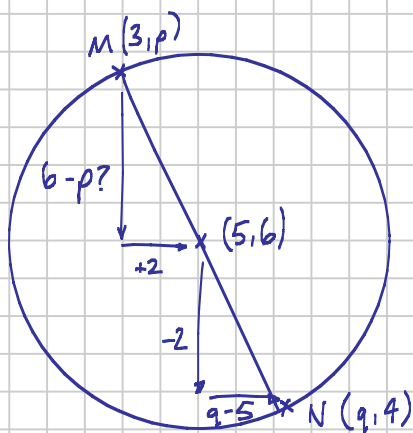


Call the centre of the circle  $M$  since it is the midpoint of  $CD$ .

$$\vec{DM} = \begin{pmatrix} -5a \\ 12a \end{pmatrix}$$

$$\Rightarrow C = (-7a, 17a)$$

- 9 The points  $M(3, p)$  and  $N(q, 4)$  lie on the circle centre  $(5, 6)$ . The line  $MN$  is a diameter of the circle. Find the value of  $p$  and  $q$ .



$$\text{since } \vec{MC} = \vec{CN}$$

$$6 - p = -2 \Rightarrow p = 8$$

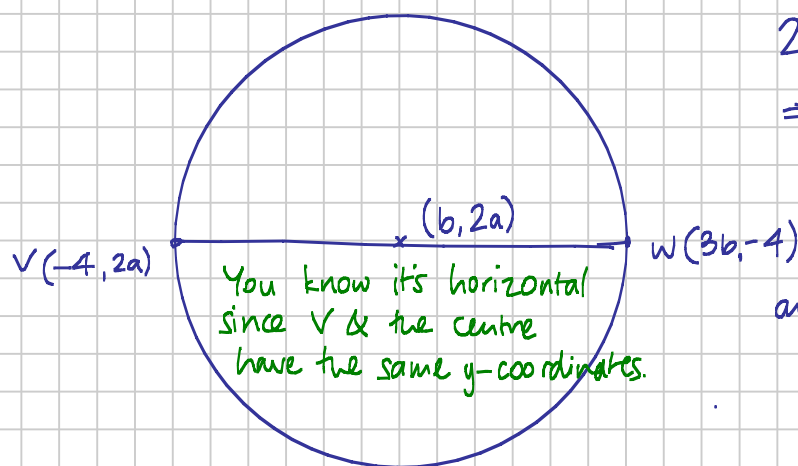
and

$$q - 5 = 2 \Rightarrow q = 7$$

$$p = 8, q = 7$$



- 10 The points  $V(-4, 2a)$  and  $W(3b, -4)$  lie on the circle centre  $(b, 2a)$ . The line  $VW$  is a diameter of the circle. Find the value of  $a$  and  $b$ .



$$2a = -4$$

$$\Rightarrow a = -2$$

$$\text{and } b - 4 = 3b - b$$

$$\Rightarrow b + 4 = 2b$$

$$\Rightarrow b = 4.$$